

Grade 8

8.1. Core Content: Linear functions and equations

(Algebra)

Students solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships, and they learn what functions are. They model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or a first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes well beyond these basic algebraic ideas.

Performance Expectations

Students are expected to:

8.1.A Solve one-variable linear equations.

8.1.B Solve one- and two-step linear inequalities and graph the solutions on the number line.

8.1.C Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.

8.1.D Determine the slope and y -intercept of a linear function described by a symbolic expression, table, or graph.

Explanatory Comments and Examples

Examples:

Solve each equation for x .

- $91 - 2.5x = 26$
- $\frac{7}{8}(x - 2) = 119$
- $-3x + 34 = 5x$
- $114 = -2x - 8 + 5x$
- $3(x - 2) - 4x = 2(x + 22) - 5$

The emphasis at this grade level is on gaining experience with inequalities, rather than on becoming proficient at solving inequalities in which multiplying or dividing by a negative is necessary.

Example:

- Graph the solution of $4x - 21 > 57$ on the number line.

Translating among these various representations of functions is an important way to demonstrate a conceptual understanding of functions.

Examples:

- Determine the slope and y -intercept for the function described by
$$y = \frac{2}{3}x + 5$$
- The following table represents a linear function. Determine the slope and y -intercept.

x	2	3	5	8	12
y	5	8	14	23	35

Performance Expectations

Students are expected to:

- 8.1.E Interpret the slope and y -intercept of the graph of a linear function representing a contextual situation.
- 8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.
- 8.1.G Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.

Explanatory Comments and Examples

Example:

- A car is traveling down a long, steep hill. The elevation, E , above sea level (in feet) of the car when it is d miles from the top of the hill is given by $E = 7500 - 250d$, where d can be any number from 0 to 6. Find the slope and y -intercept of the graph of this function and explain what they mean in the context of the moving car.

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Example:

- Mike and Tim leave their houses at the same time to walk to school. Mike's walk can be represented by $d_1 = 4000 - 400t$, and Tim's walk can be represented by $d_2 = 3400 - 250t$, where d is the distance from the school in feet and t is the walking time in minutes. Who arrives at school first? By how many minutes? Is there a time when Mike and Tim are the same distance away from the school? Explain your reasoning.

Examples:

- Could the data presented in the table represent a linear function? Explain your reasoning.

x	-1	0	1	2	3	4	5
y	0	-1	0	3	8	15	24

- Does $y = \frac{1}{4}x - 5$ represent a linear function? Explain your reasoning.

Grade 8

8.2. Core Content: Properties of geometric figures

(Numbers, Geometry/Measurement)

Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships involving sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will further study in high school.

Performance Expectations

Students are expected to:

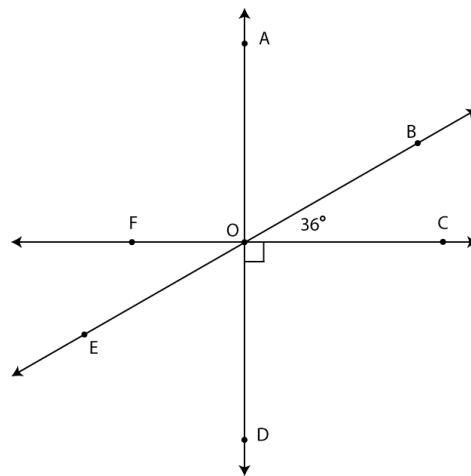
- 8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.

- 8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.

Explanatory Comments and Examples

Example:

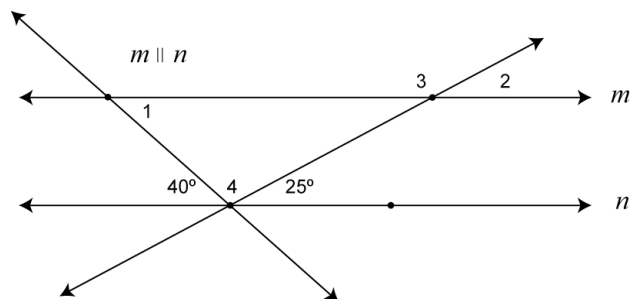
- Determine the measures of $\angle BOA$, $\angle EOD$, $\angle FOB$, and $\angle FOE$ and explain how you found each measure. As part of your explanation, identify pairs of angles as complementary, supplementary, or vertical.



Example:

- Determine the measures of the indicated angles.

$\angle 1$: _____ $\angle 2$: _____ $\angle 3$: _____ $\angle 4$: _____



Performance Expectations

Students are expected to:

8.2.C Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.

8.2.D Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.

8.2.E Quickly recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.

8.2.F Demonstrate the Pythagorean Theorem and its converse and apply them to solve problems.

8.2.G Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.

Explanatory Comments and Examples

Examples:

- Determine the measure of each interior angle in a regular pentagon.
- In a certain triangle, the measure of one angle is four times the measure of the smallest angle, and the measure of the remaining angle is the sum of the measures of the other two angles. Determine the measure of each angle.

Example:

- Consider a trapezoid with vertices (1,2), (1,6), (6,4), and (6,2). The trapezoid is reflected across the x -axis and then translated four units to the left. Graph the image of the trapezoid after these two transformations and give the coordinates of the new vertices.

Students can use perfect squares of integers to determine squares and square roots of related numbers, such as 1.96 and 0.0049.

Examples:

- Determine: $\sqrt{36}$, $\sqrt{0.25}$, $\sqrt{144}$, and $\sqrt{196}$.
- Between which two consecutive integers does the square root of 74 lie?

One possible demonstration is to start with a right triangle, use each of the three triangle sides to form the side of a square, and draw the remaining three sides of each of the three squares. The areas of the three squares represent the Pythagorean relationship.

Examples:

- Is a triangle with side lengths 5 cm, 12 cm, and 13 cm a right triangle? Why or why not?
- Determine the length of the diagonal of a rectangle that is 7 ft by 10 ft.

Example:

- Determine the distance between the points $(-2, 3)$ and $(4, 7)$.

Grade 8

8.3. Core Content: Summary and analysis of data sets (Algebra, Data/Statistics/Probability)

Students build on their extensive experience organizing and interpreting data and apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in newspapers, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

Performance Expectations

Students are expected to:

- 8.3.A Summarize and compare data sets in terms of variability and measures of center.

- 8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.

Explanatory Comments and Examples

Students use mean, median, mode, range, and interquartile range to summarize and compare data sets, and explain the influence of outliers on each measure.

Example:

- Captain Bob owns two charter boats, the *Sock-Eye-To-Me* and *Old Gus*, which take tourists on fishing trips. On Saturday, the *Sock-Eye-To-Me* took four people fishing and returned with eight fish weighing 18, 23, 20, 6, 20, 22, 18, and 20 pounds. On the same day, *Old Gus* took five people fishing and returned with ten fish weighing 38, 18, 12, 14, 17, 42, 12, 16, 12, and 14 pounds.

Using measures of center and variability, compare the weights of the fish caught by the people in the two boats.

Make a summary statement telling which boat you would charter for fishing based on these data and why.

What influence, if any, do outliers have on the particular statistics for these data?

Previously studied displays include stem-and-leaf plots, histograms, circle graphs, and line plots. Here these displays are used to compare data sets. Box-and-whisker plots are introduced here for the first time as a powerful tool for comparing two or more data sets.

Example:

- As part of their band class, Tayla and Alyssa are required to keep practice records that show the number of minutes they practice their instruments each day. Below are their practice records for the past fourteen days:

Performance Expectations

Students are expected to:

- 8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.

Explanatory Comments and Examples

Tayla: 55, 45, 60, 45, 30, 30, 90, 50, 40, 75, 25, 90, 105, 60

Alyssa: 20, 120, 25, 20, 0, 15, 30, 15, 90, 0, 30, 30, 10, 30

Of stem-and-leaf plot, circle graph, or line plot, select the data display that you think will best compare the two girls' practice records.

Construct a display to show the data.

Compare the amount of time the two girls practice by analyzing the data presented in the display.

Example:

- Kera randomly selected seventeen students from her middle school for a study comparing arm span to standing height. The students' measurements are shown in the table below.

Comparison of Arm Span and Standing Height (in cm) at Icicle River Middle School

Height (cm)	Arm Span (cm)	Height (cm)	Arm Span (cm)
138	145	155	150
135	135	175	177
142	147	162	160
158	145	150	152
177	174	142	143
150	152	149	149
158	162	160	165
160	160	173	170
160	158		

Create a scatterplot for the data shown.

If appropriate, sketch a trendline.

Use these data to estimate the arm span of a student with a height of 180 cm, and the height of a student with an arm span of 130 cm. Explain any limitations of using this process to make estimates.

Performance Expectations

Students are expected to:

8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.

8.3.E Determine whether conclusions of statistical studies reported in the media are reasonable.

8.3.F Determine probabilities for mutually exclusive, dependent, and independent events for small sample spaces.

Explanatory Comments and Examples

Students should work with a variety of sampling techniques and should be able to identify strengths and weakness of random, census, convenience, and representative sampling.

Example:

- Carli, Jamar, and Amberly are conducting a survey to determine their school's favorite Seattle professional sports team. Carli selects her sample using a convenience method—she surveys students on her bus during the ride to school. Jamar uses a computer to randomly select 30 numbers from 1 through 600, and then surveys the corresponding students from a numbered, alphabetical listing of the student body. Amberly waits at the front entrance before school and surveys every twentieth student entering. Analyze the strengths and weaknesses of each method.

Examples:

- Given a standard deck of 52 playing cards, what is the probability of drawing a king or queen? [Some students may be unfamiliar with playing cards, so alternate examples may be desirable.]
- Leyanne is playing a game at a birthday party. Beneath ten paper cups, a total of five pieces of candy are hidden, with one piece hidden beneath each of five cups. Given only three guesses, Leyanne must uncover three pieces of candy to win all the hidden candy. What is the probability she will win all the candy?
- A bag contains 7 red marbles, 5 blue marbles, and 8 green marbles. If one marble is drawn at random and put back in the bag, and then a second marble is drawn at random, what is the probability of drawing first a red marble, then a blue marble?

Performance Expectations

Students are expected to:

- 8.3.G Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions.

Explanatory Comments and Examples

The intent of this expectation is for students to show their work, explain their thinking, and verify that the answer to the problem is reasonable in terms of the original context and the mathematics used to solve the problem. Verifications can include the use of numbers, words, pictures, or equations.

Counting techniques include the fundamental counting principle, lists, tables, tree diagrams, etc.

Examples:

- Jack's Deli makes sandwiches that include a choice of one type of bread, one type of cheese, and one type of meat. How many different sandwiches could be made given 4 different bread types, 3 different cheeses, and 5 different meats? Explain your reasoning.
- A small high school has 57 tenth-graders. Of these students, 28 are taking geometry, 34 are taking biology, and 10 are taking neither geometry nor biology. How many students are taking both geometry and biology? How many students are taking geometry but not biology? How many students are taking biology but not geometry? Draw a Venn diagram to illustrate this situation.

Grade 8

8.4. Additional Key Content

(Numbers, Operations)

Students deal with a few key topics about numbers as they prepare to shift to higher level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become especially important as “extreme units” continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables. Developing this skill extends students’ work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real-number system, students are introduced to simple irrational numbers, thus preparing them to study higher level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Performance Expectations

Students are expected to:

8.4.A Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.

8.4.B Solve problems involving operations with numbers in scientific notation and verify solutions.

Explanatory Comments and Examples

Examples:

- Represent 4.27×10^{-3} in standard form.
- Represent 18,300,000 in scientific notation.
- Throughout the year 2004, people in the city of Cantonville sent an average of 400 million text messages a day. Using this information, about how many total text messages did Cantonville residents send in 2004? (2004 was a leap year.) Express your answer in scientific notation.

Units include those associated with technology, such as nanoseconds, gigahertz, kilobytes, teraflops, etc.

Examples:

- A supercomputer used by a government agency will be upgraded to perform 256 teraflops (that is, 256 trillion calculations per second). Before the upgrade, the supercomputer performs 1.6×10^{13} calculations per second. How many more calculations per second will the upgraded supercomputer be able to perform? Express the answer in scientific notation.
- A nanosecond is one billionth of a second. How many nanoseconds are there in five minutes? Express the answer in scientific notation.

Performance Expectations

Students are expected to:

- 8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.

- 8.4.D Identify rational and irrational numbers.

Explanatory Comments and Examples

Example:

- Simplify and write the answer in exponential form:

$$\frac{(7^4)^2}{7^3}$$

Some students will be ready to solve problems involving simple negative exponents and should be given the opportunity to do so.

Example:

- Simplify and write the answer in exponential form:

$$(5^4)^2 5^{-3}$$

Students should know that rational numbers are numbers that can be represented as the ratio of two integers; that the decimal expansions of rational numbers have repeating patterns, or *terminate*; and that there are numbers that are not rational.

Example:

- Identify whether each number is rational or irrational and explain your choice.

$$3.14, 4.\bar{6}, \frac{1}{11}, \sqrt{2}, \sqrt{25}, \pi$$

Grade 8

8.5. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Performance Expectations

Students are expected to:

- 8.5.A Analyze a problem situation to determine the question(s) to be answered.
- 8.5.B Identify relevant, missing, and extraneous information related to the solution to a problem.
- 8.5.C Analyze and compare mathematical strategies for solving problems, and select and use one or more strategies to solve a problem.
- 8.5.D Represent a problem situation, describe the process used to solve the problem, and verify the reasonableness of the solution.
- 8.5.E Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.
- 8.5.F Apply a previously used problem-solving strategy in a new context.
- 8.5.G Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.
- 8.5.H Make and test conjectures based on data (or information) collected from explorations and experiments.

Explanatory Comments and Examples

Descriptions of solution processes and explanations can include numbers, words (including mathematical language), pictures, or equations. Students should be able to use all of these representations as needed. For a particular solution, students should be able to explain or show their work using at least one of these representations and verify that their answer is reasonable.

Examples:

- The dimensions of a room are 12 feet by 15 feet by 10 feet. What is the furthest distance between any two points in the room? Explain your solution.
- Miranda's phone service contract ends this month. She is looking for ways to save money and is considering changing phone companies. Her current cell phone carrier, X-Cell, calculates the monthly bill using the equation $c = \$15.00 + \$0.07m$, where c represents the total monthly cost and m represents the number of minutes of talk time during a monthly billing cycle. Another company, Prism Cell, offers 300 free minutes of talk time each month for a base fee of \$30.00 with an additional \$0.15 for every minute over 300 minutes. Miranda's last five phone bills were \$34.95, \$36.70, \$37.82, \$62.18, and \$36.28. Using the data from the last five months, help Miranda decide whether she should switch companies. Justify your answer.